Phys 410

Fall 2014

Homework #0

Due Thursday, 4 September, 2014

Complete the following by hand (no assistance from computers!):

1. Use the Euler formula to expand $e^{i\theta}$ for real θ.
2. Given the three Cartesian unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$, calculate the following:
    a. $\hat{x} \times \hat{y}$
    b. $|\hat{x}|$
    c. $\hat{x} \cdot \hat{y}$
3. Given the vectors $\vec{r} = (r_x, r_y, r_z)$ and $\vec{s} = (s_x, s_y, s_z)$, calculate the cross product vector $\vec{r} \times \vec{s}$ in terms of its Cartesian components.
4. Find the eigenvalues and eigenvectors of this matrix: $\bar{\bar{A}} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.
5. What is the determinant of $\bar{\bar{A}}$ and how is it related to the eigenvalues?
6. What is the trace of $\bar{\bar{A}}$ and how is it related to the eigenvalues?
7. Given a scalar function of position $\chi(\vec{r})$ (e.g. the temperature distribution on the surface of the earth), what can we say is always true about the curl of the gradient of χ?
8. Given a vector field $\vec{F} = k(x, 2y^2, 3z^3)$, where k is a constant, calculate its curl, $\nabla \times \vec{F}$.
9. What is the general solution to the second-order linear differential equation $\ddot{x} = -\omega^2 x$, where ω is a real positive number?
10. What is the general solution to the second-order linear differential equation $\ddot{x} = +k^2 x$, where k is a real positive number?
11. Given $ln(y) = b\ ln(x)$, where b is a constant, find y as a function of x, $y(x)$.
12. Evaluate the integral $I = \int_{-2}^{3} 5x\ dx$.
13. Expand $y(x) = \ln(1+x)$ to second order for $x \ll 1$. Write the series expansion for $y(x) = \frac{1}{1-x}$ valid for $-1 < x < 1$.
14. Write down the differential volume element d^3r in spherical coordinates. Use the figure below for definition of the spherical coordinates.